040411

$0<\theta<\dfrac{\pi}{2}$ $\dfrac{4}{\sin\theta}+\dfrac{9}{\cos\theta}$

$0<\theta<\dfrac{\pi}{2}$ $\dfrac{4}{\sin\theta}+\dfrac{9}{\cos\theta}$ $\dfrac{a}{\sin^n\theta}+\dfrac{b}{\cos^n\theta}$

($a>0$, $b>0$, $n\in\mathbf{N}$) n

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$a_1,\ a_2\ ,\cdots,\ a_n\in\mathbb{R}$; $b_1,\ b_2,\cdots,\ b_n\in\mathbb{R}$

$(a_1{}^2+a_2{}^2+\cdots+a_n{}^2)(b_1{}^2+b_2{}^2+\cdots+b_n{}^2)\geq(a_1b_1+a_2b_2+\cdots+a_nb_n)^2$ ……①

$\Leftrightarrow$ $a_1=kb_1$, $a_2=kb_2$, $\cdots$, $a_n=kb_n$, $k\in\mathbb{R}$

$A=(\sum_{k=1}^{n}a_k{}^2)^{\frac{1}{2}},B=(\sum_{k=1}^{n}b_k{}^2)^{\frac{1}{2}}$ $\sum_{k=1}^{n}\dfrac{a_k{}^2}{A^2}=\sum_{k=1}^{n}\dfrac{b_k{}^2}{B^2}=1$

$$2=\sum_{k=1}^{n}(\frac{a_k{}^2}{A^2}+\frac{b_k{}^2}{B^2})\geq\sum_{k=1}^{n}\frac{2|a_kb_k|}{AB}$$

$AB \ge \sum_{k=1}^{n} |a_k b_k|$ $\sum_{k=1}^{n} |a_k b_k| \ge \left|\sum_{k=1}^{n} a_k b_k\right|$ $AB \ge \sum_{k=1}^{n} a_k b_k$ $A^2B^2 \ge (\sum_{k=1}^{n} a_k b_k)^2$

①

$$\Leftrightarrow \quad \frac{a_1^{\ 2}}{A^2} = \frac{b_1^{\ 2}}{B^2} \Rightarrow a_1 : b_1 = A : B \ (\because a_1, b_1 \text{ 同號})$$

$$\frac{a_2^{\ 2}}{A^2} = \frac{b_2^{\ 2}}{B^2} \Rightarrow a_2 : b_2 = A : B \ (\because a_2, b_2 \text{ 同號})$$

$$\frac{a_3^{\ 2}}{A^2} = \frac{b_3^{\ 2}}{B^2} \Rightarrow a_3 : b_3 = A : B \ (\because a_3, b_3 \text{ 同號})$$

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$$\frac{a_n^{\ 2}}{A} = \frac{b_n^{\ 2}}{B} \Rightarrow a_n : b_n = A : B \ (\because a_n, b_n \text{ 同號})$$

$a_1 : b_1 = a_2 : b_2 = a_3 : b_3 = \cdots = a_n : b_n$ $a_i = 0$ $b_i = 0$

$a_1 = kb_1$, $a_2 = kb_2$, $\cdots$, $a_n = kb_n$, $k \in \mathbb{R}$

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1. . $(\sum_{k=1}^{n} a_k b_k)(\sum_{k=1}^{n} \frac{a_k}{b_k}) \ge (\sum_{k=1}^{n} a_k)^2 \quad (a_k > 0, b_k > 0)$

2. . $(\sum_{k=1}^{n} a_k)(\sum_{k=1}^{n} b_k) \ge (\sum_{k=1}^{n} \sqrt{a_k b_k})^2 \quad (a_k > 0, b_k > 0)$

3. . $(\sum_{k=1}^{n} a_k)(\sum_{k=1}^{n} \frac{b_k^{\ 2}}{a_k}) \ge (\sum_{k=1}^{n} b_k)^2 \quad (a_k > 0)$

4. . $(\sum_{k=1}^{n} \frac{b_k^{\ 2}}{a_k}) \ge (\sum_{k=1}^{n} b_k)^2 / (\sum_{k=1}^{n} a_k) \quad (a_k > 0)$

5. . $(\sum_{k=1}^{n} \frac{x_k^{\ 2}}{y_k + z_k}) \ge (\sum_{k=1}^{n} x_k)^2 / [\sum_{k=1}^{n} (y_k + z_k)] \quad (y_k > 0, z_k > 0)$

1. $(\sum_{k=1}^{n} x_k^{\ 2})(\sum_{k=1}^{n} y_k^{\ 2}) \ge (\sum_{k=1}^{n} x_k y_k)^2 \cdots\cdots$ ②

$x_k^{\ 2} = a_k b_k$, $y_k^{\ 2} = \frac{a_k}{b_k}$ ②

2. $x_k^{\ 2} = a_k$, $y_k^{\ 2} = b_k$ ②

3. $x_k^{\ 2} = a_k$, $y_k^{\ 2} = \frac{b_k^{\ 2}}{a_k}$ ②

4. 3

5. 4 $b_k = x_k$, $a_k = y_k + z_k$

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m $< a_{11}, a_{12}, \cdots, a_{1n} >, < a_{21}, a_{22}, \cdots, a_{2n} >, \cdots, < a_{m1}, a_{m2}, \cdots, a_{mn} >$

$$(a_{11}{}^m + a_{12}{}^m + \cdots + a_{1n}{}^m)(a_{21}{}^m + a_{22}{}^m + \cdots + a_{2n}{}^m)\cdots(a_{m1}{}^m + a_{m2}{}^m + \cdots + a_{mn}{}^m) \geq$$

$$(a_{11}a_{21}\cdots a_{m1} + a_{12}a_{22}\cdots a_{m2} + \cdots + a_{1n}a_{2n}\cdots a_{mn})^m$$

$$\frac{a_{11}{}^m}{\sum_{i=1}^n a_{1i}{}^m} + \frac{a_{21}{}^m}{\sum_{i=1}^n a_{2i}{}^m} + \frac{a_{31}{}^m}{\sum_{i=1}^n a_{3i}{}^m} + \cdots + \frac{a_{m1}{}^m}{\sum_{i=1}^n a_{mi}{}^m} \geq m \cdot \frac{|a_{11}a_{21}a_{31}\cdots a_{m1}|}{\sqrt[m]{\sum_{i=1}^n a_{1i}{}^n \sum_{i=1}^n a_{2i}{}^n \sum_{i=1}^n a_{3i}{}^n \cdots \sum_{i=1}^n a_{mi}{}^n}}$$

$$\frac{a_{12}{}^m}{\sum_{i=1}^n a_{1i}{}^m} + \frac{a_{22}{}^m}{\sum_{i=1}^n a_{2i}{}^m} + \frac{a_{32}{}^m}{\sum_{i=1}^n a_{3i}{}^m} + \cdots + \frac{a_{m2}{}^m}{\sum_{i=1}^n a_{mi}{}^m} \geq m \cdot \frac{|a_{12}a_{22}a_{32}\cdots a_{m2}|}{\sqrt[m]{\sum_{i=1}^n a_{1i}{}^n \sum_{i=1}^n a_{2i}{}^n \sum_{i=1}^n a_{3i}{}^n \cdots \sum_{i=1}^n a_{mi}{}^n}}$$

……………………………

$$\frac{a_{1n}{}^m}{\sum_{i=1}^n a_{1i}{}^m} + \frac{a_{2n}{}^m}{\sum_{i=1}^n a_{2i}{}^m} + \frac{a_{3n}{}^m}{\sum_{i=1}^n a_{3i}{}^m} + \cdots + \frac{a_{mn}{}^m}{\sum_{i=1}^n a_{mi}{}^m} \geq m \cdot \frac{|a_{1n}a_{2n}a_{3n}\cdots a_{mn}|}{\sqrt[m]{\sum_{i=1}^n a_{1i}{}^n \sum_{i=1}^n a_{2i}{}^n \sum_{i=1}^n a_{3i}{}^n \cdots \sum_{i=1}^n a_{mi}{}^n}}$$

$$m \geq m \cdot \frac{|a_{11}a_{21}a_{31}\cdots a_{n1} + a_{12}a_{22}a_{32}\cdots a_{n2} + \cdots + a_{n1}a_{n2}a_{n3}\cdots a_{nm}|}{\sqrt[m]{\sum_{i=1}^n a_{1i}{}^m \sum_{i=1}^n a_{2i}{}^m \sum_{i=1}^n a_{3i}{}^m \cdots \sum_{i=1}^n a_{ni}{}^m}}$$

m

$$\sum_{i=1}^n a_{1i}{}^m \sum_{i=1}^n a_{2i}{}^m \sum_{i=1}^n a_{3i}{}^m \cdots \sum_{i=1}^n a_{ni}{}^m \geq (|a_{11}a_{21}a_{31}\cdots a_{m1} + a_{12}a_{22}a_{32}\cdots a_{m2} + \cdots + a_{n1}a_{n2}a_{n3}\cdots a_{mn}|)^m \geq (\sum_{i=1}^n a_{1i}a_{2i}a_{3i}\cdots a_{mi})^m$$

( ) $0 < \theta < \frac{\pi}{2}$ $\frac{a}{\sin^n\theta} + \frac{b}{\cos^n\theta}$ ($a > 0$, $b > 0$, $n \in \mathbf{N}$)

1. $n = 1$ $0 < \theta < \frac{\pi}{2}$ $\frac{4}{\sin\theta} + \frac{9}{\cos\theta}$

$$[(\sqrt[3]{\frac{4}{\sin\theta}})^3 + (\sqrt[3]{\frac{9}{\cos\theta}})^3]^2 \cdot [(\sqrt[3]{\sin^2\theta})^3 + (\sqrt[3]{\cos^2\theta})^3]^1$$

$$\geq [(\sqrt[3]{\frac{4}{\sin\theta}})^2 \cdot \sqrt[3]{\sin^2\theta} + (\sqrt[3]{\frac{9}{\cos\theta}})^2 \cdot \sqrt[3]{\cos^2\theta}]^3 = (4^{\frac{2}{3}} + 9^{\frac{2}{3}})^3$$

$\frac{4}{\sin\theta} + \frac{9}{\cos\theta}$ $(4^{\frac{2}{3}} + 9^{\frac{2}{3}})^{\frac{3}{2}}$

$$\sqrt[3]{\frac{4}{\sin\theta}}:\sqrt[3]{\frac{9}{\cos\theta}}=\sqrt[3]{\sin^2\theta}:\sqrt[3]{\cos^2\theta} \qquad \sin\theta:\cos\theta=\sqrt[3]{4}:\sqrt[3]{9}$$

2. $n=2$ $\quad 0<\theta<\frac{\pi}{2}$ $\quad \frac{4}{\sin^2\theta}+\frac{9}{\cos^2\theta}$

$$[(\sqrt{\frac{4}{\sin^2\theta}})^2+(\sqrt{\frac{9}{\cos^2\theta}})^2]^2\cdot[(\sin^2\theta+\cos^2\theta]\ge(\frac{2}{\sin\theta}\cdot\sin\theta+\frac{3}{\cos\theta}\cdot\cos\theta)^2=(2+3)^2=25$$

$\frac{4}{\sin^2\theta}+\frac{9}{\cos^2\theta}$ $\quad 25$

$$\sqrt{\frac{4}{\sin^2\theta}}:\sqrt{\frac{9}{\cos^2\theta}}=\sin\theta:\cos\theta \qquad \sin\theta:\cos\theta=\sqrt{2}:\sqrt{3}$$

3. $n=3$ $\quad 0<\theta<\frac{\pi}{2}$ $\quad \frac{a}{\sin^3\theta}+\frac{b}{\cos^3\theta}$

$$[(\sqrt[5]{\frac{a}{\sin^3\theta}})^5+(\sqrt[5]{\frac{b}{\cos^3\theta}})^5]^2\cdot[(\sqrt[5]{\sin^2\theta})^5+(\sqrt[5]{\cos^2\theta})^5]^3$$

$$\ge[(\sqrt[5]{\frac{a}{\sin^3\theta}})^2\cdot(\sqrt[5]{\sin^2\theta})^3+(\sqrt[5]{\frac{b}{\cos^3\theta}})^2\cdot(\sqrt[5]{\cos^2\theta})^3]^5=(a^{\frac{2}{5}}+b^{\frac{2}{5}})^5$$

$\frac{a}{\sin^3\theta}+\frac{b}{\cos^3\theta}$ $\quad (a^{\frac{2}{5}}+b^{\frac{2}{5}})^{\frac{5}{2}}$

$$\sqrt[5]{\frac{a}{\sin^3\theta}}:\sqrt[5]{\frac{b}{\cos^3\theta}}=\sqrt[5]{\sin^2\theta}:\sqrt[5]{\cos^2\theta} \qquad \sin\theta:\cos\theta=\sqrt[5]{a}:\sqrt[5]{b}$$

4. $\quad 0<\theta<\frac{\pi}{2}$ $\quad \frac{a}{\sin^n\theta}+\frac{b}{\cos^n\theta}$

$(-n)\times2+2\times n=0$ $\quad 2 \quad (\sqrt[n+2]{\frac{a}{\sin^n\theta}})^{n+2}+(\sqrt[n+2]{\frac{b}{\cos^n\theta}})^{n+2}$ $\quad n$

$(\sqrt[n+2]{\sin^2\theta})^{n+2}+(\sqrt[n+2]{\cos^2\theta})^{n+2}$

$$[(\sqrt[n+2]{\frac{a}{\sin^n\theta}})^{n+2}+(\sqrt[n+2]{\frac{b}{\cos^n\theta}})^{n+2}]^2\cdot[(\sqrt[n+2]{\sin^2\theta})^{n+2}+(\sqrt[n+2]{\cos^2\theta})^{n+2}]^n$$

$$\ge[(\sqrt[n+2]{\frac{a}{\sin^n\theta}})^2\cdot(\sqrt[n+2]{\sin^2\theta})^n+(\sqrt[n+2]{\frac{b}{\cos^n\theta}})^2\cdot(\sqrt[n+2]{\cos^2\theta})^n]^{n+2}=(a^{\frac{2}{n+2}}+b^{\frac{2}{n+2}})^{n+2}$$

$\frac{a}{\sin^n\theta}+\frac{b}{\cos^n\theta}$ $\quad (a^{\frac{2}{n+2}}+b^{\frac{2}{n+2}})^{\frac{n+2}{2}}$

$$\sqrt[n+2]{\frac{a}{\sin^n\theta}}:\sqrt[n+2]{\frac{b}{\cos^n\theta}}=\sqrt[n+2]{\sin^2\theta}:\sqrt[n+2]{\cos^2\theta} \qquad \sin\theta:\cos\theta=\sqrt[n+2]{a}:\sqrt[n+2]{b}$$

( ) $0<\theta<\frac{\pi}{2}$ $\frac{a}{\sin^n\theta}+\frac{b}{\cos^n\theta}$ ($a>0$, $b>0$, $n\in\mathbf{Q}^+$)

1. $0<\theta<\frac{\pi}{2}$ $\frac{a}{\sin^{\frac{1}{2}}\theta}+\frac{b}{\cos^{\frac{1}{2}}\theta}$ ($a>0$, $b>0$)

$(-\frac{1}{2})\times 4+2\times 1=0$ 4 $(\sqrt[5]{\frac{a}{\sin^{\frac{1}{2}}\theta}})^5+(\sqrt[5]{\frac{b}{\cos^{\frac{1}{2}}\theta}})^5$ 1 $(\sqrt[5]{\sin^2\theta})^5+(\sqrt[5]{\cos^2\theta})^5$

$$[(\sqrt[5]{\frac{a}{\sin^{\frac{1}{2}}\theta}})^5+(\sqrt[5]{\frac{b}{\cos^{\frac{1}{2}}\theta}})^5]^4\cdot[(\sqrt[5]{\sin^2\theta})^5+(\sqrt[5]{\cos^2\theta})^5]^1$$
$$\ge[(\sqrt[5]{\frac{a}{\sin^{\frac{1}{2}}\theta}})^4\cdot\sqrt[5]{\sin^2\theta}+(\sqrt[5]{\frac{b}{\cos^{\frac{1}{2}}\theta}})^4\cdot\sqrt[5]{\cos^2\theta}]^5=(a^{\frac{4}{5}}+b^{\frac{4}{5}})^5$$

$\frac{a}{\sin^{\frac{1}{2}}\theta}+\frac{b}{\cos^{\frac{1}{2}}\theta}$ $(a^{\frac{4}{5}}+b^{\frac{4}{5}})^{\frac{5}{4}}$

$$\sqrt[5]{\frac{a}{\sin^{\frac{1}{2}}\theta}}:\sqrt[5]{\frac{b}{\cos^{\frac{1}{2}}\theta}}=\sqrt[5]{\sin^2\theta}:\sqrt[5]{\cos^2\theta} \qquad \sin\theta:\cos\theta=\sqrt[5]{a^2}:\sqrt[5]{b^2}$$

2. $n=\frac{q}{p}\in\mathbb{Q}$, $p,q\in\mathbb{N}$ $0<\theta<\frac{\pi}{2}$ $\frac{a}{\sin^{\frac{q}{p}}\theta}+\frac{b}{\cos^{\frac{q}{p}}\theta}$ ($a>0$, $b>0$)

$\frac{q}{p}:2=q:2p$ $(-\frac{q}{p})\times 2p+2\times q=0$

$2p$ $(\sqrt[2p+q]{\frac{a}{\sin^{\frac{q}{p}}\theta}})^{2p+q}+(\sqrt[2p+q]{\frac{b}{\cos^{\frac{q}{p}}\theta}})^{2p+q}$ q $(\sqrt[2p+q]{\sin^2\theta})^{2p+q}+(\sqrt[2p+q]{\cos^2\theta})^{2p+q}$

$$[(\sqrt[2p+q]{\frac{a}{\sin^{\frac{q}{p}}\theta}})^{2p+q}+(\sqrt[2p+q]{\frac{b}{\cos^{\frac{q}{p}}\theta}})^{2p+q}]^{2p}\cdot[(\sqrt[2p+q]{\sin^2\theta})^{2p+q}+(\sqrt[2p+q]{\cos^2\theta})^{2p+q}]^q$$
$$\ge[(\sqrt[2p+q]{\frac{a}{\sin^{\frac{q}{p}}\theta}})^{2p}\cdot(\sqrt[2p+q]{\sin^2\theta})^q+(\sqrt[2p+q]{\frac{b}{\cos^{\frac{q}{p}}\theta}})^{2p}\cdot(\sqrt[2p+q]{\cos^2\theta})^q]^{2p+q}=(a^{\frac{2p}{2p+q}}+b^{\frac{2p}{2p+q}})^{2p+q}$$

$$\frac{a}{\sin^{\frac{q}{p}}\theta}+\frac{b}{\cos^{\frac{q}{p}}\theta} \qquad (a^{\frac{2p}{2p+q}}+b^{\frac{2p}{2p+q}})^{\frac{2p+q}{2p}}$$

$$\sqrt[2p+q]{\frac{a}{\sin^{\frac{q}{p}}\theta}}:\sqrt[2p+q]{\frac{b}{\cos^{\frac{q}{p}}\theta}}=\sqrt[2p+q]{\sin^2\theta}:\sqrt[2p+q]{\cos^2\theta} \qquad \sin\theta:\cos\theta=\sqrt[2p+q]{a^p}:\sqrt[2p+q]{b^p}$$

( ) $x^k+y^k+z^k=1$

$\frac{a}{x^{\frac{q}{p}}}+\frac{b}{y^{\frac{q}{p}}}+\frac{c}{z^{\frac{q}{p}}}$ ($x>0$, $y>0$, $z>0$, $a>0,\ b>0, c>0$, $k,p,q\in\mathbf{N}$)

$$\frac{q}{p}:k=q:pk \qquad (-\frac{q}{p})\times pk+k\times q=0$$

$$pk \quad [(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk+q}] \quad q$$

$$[(\sqrt[pk+q]{x^k})^{pk+q}+(\sqrt[pk+q]{y^k})^{pk+q}+(\sqrt[pk+q]{z^k})^{pk+q}]$$

$$[(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk+q}]^{pk}\cdot[(\sqrt[pk+q]{x^k})^{pk+q}+(\sqrt[pk+q]{y^k})^{pk+q}+(\sqrt[pk+q]{z^k})^{pk+q}]^{q}$$

$$\geq[(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{x^k})^{q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{y^k})^{q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{z^k})^{q}]^{pk+q}=(a^{\frac{pk}{pk+q}}+b^{\frac{pk}{pk+q}}+c^{\frac{pk}{pk+q}})^{pk+q}$$

$$\frac{a}{x^{\frac{q}{p}}}+\frac{b}{y^{\frac{q}{p}}}+\frac{c}{z^{\frac{q}{p}}} \qquad (a^{\frac{pk}{pk+q}}+b^{\frac{pk}{pk+q}}+c^{\frac{pk}{pk+q}})^{\frac{pk+q}{pk}}$$

$$\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}}:\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}}:\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}}=\sqrt[pk+q]{x^k}:\sqrt[pk+q]{y^k}:\sqrt[pk+q]{z^k} \qquad x:y:z=\sqrt[pk+q]{a^p}:\sqrt[pk+q]{b^p}:\sqrt[pk+q]{c^p}$$

( ) $x^k+y^k+z^k=t\,(t>0)$

$\frac{a}{x^{\frac{q}{p}}}+\frac{b}{y^{\frac{q}{p}}}+\frac{c}{z^{\frac{q}{p}}}$ ($x>0$, $y>0$, $z>0$, $a>0,\ b>0, c>0$, $p,q\in\mathbf{N}$)

$$\frac{q}{p}:k=q:pk \qquad (-\frac{q}{p})\times pk+k\times q=0$$

pk $[(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk+q}]$ q

$[(\sqrt[pk+q]{x^k})^{pk+q}+(\sqrt[pk+q]{y^k})^{pk+q}+(\sqrt[pk+q]{z^k})^{pk+q}]$

$$[(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk+q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk+q}]^{pk}\cdot[(\sqrt[pk+q]{x^k})^{pk+q}+(\sqrt[pk+q]{y^k})^{pk+q}+(\sqrt[pk+q]{z^k})^{pk+q}]^{q}$$

$$\geq[(\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{x^k})^{q}+(\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{y^k})^{q}+(\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}})^{pk}\cdot(\sqrt[pk+q]{z^k})^{q}]^{pk+q}=(a^{\frac{pk}{pk+q}}+b^{\frac{pk}{pk+q}}+c^{\frac{pk}{pk+q}})^{pk+q}$$

$\frac{a}{x^{\frac{q}{p}}}+\frac{b}{y^{\frac{q}{p}}}+\frac{c}{z^{\frac{q}{p}}}$ $\frac{1}{t}\cdot(a^{\frac{pk}{pk+q}}+b^{\frac{pk}{pk+q}}+c^{\frac{pk}{pk+q}})^{\frac{pk+q}{pk}}$ …………(A)

$\sqrt[pk+q]{\frac{a}{x^{\frac{q}{p}}}}:\sqrt[pk+q]{\frac{b}{y^{\frac{q}{p}}}}:\sqrt[pk+q]{\frac{c}{z^{\frac{q}{p}}}}=\sqrt[pk+q]{x^k}:\sqrt[pk+q]{y^k}:\sqrt[pk+q]{z^k}$ $x:y:z=\sqrt[pk+q]{a^p}:\sqrt[pk+q]{b^p}:\sqrt[pk+q]{c^p}$

( ) $ax+by+cz=k\ (k>0)$

$x^n+y^n+z^n\ (x>0,\ y>0,\ z>0,\ a>0,\ b>0, c>0,\ n\in\mathbf{N})$

1 $(x^n+y^n+z^n)$ $n-1$ $(\sqrt[n-1]{a})^n+(\sqrt[n-1]{b})^n+(\sqrt[n-1]{c})^n$

$(x^n+y^n+z^n)^1\cdot[(\sqrt[n-1]{a})^n+(\sqrt[n-1]{b})^n+(\sqrt[n-1]{c})^n]^{n-1}\geq[x\cdot(\sqrt[n-1]{a})^{n-1}+y\cdot(\sqrt[n-1]{b})^{n-1}+z\cdot(\sqrt[n-1]{c})^{n-1}]^n=k^n$ ………(A)

$x^n+y^n+z^n$ $\frac{k^n}{[(\sqrt[n-1]{a})^n+(\sqrt[n-1]{b})^n+(\sqrt[n-1]{c})^n]^{n-1}}$

$\Leftrightarrow\ x:y:z=(\sqrt[n-1]{a})^n:(\sqrt[n-1]{b})^n:(\sqrt[n-1]{c})^n$

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1. x y $x+y=3$ x^3+y^3

$k=3,\ n=3,\ a=1,\ b=1$ (A) x^3+y^3 $(\frac{1}{2})^2\cdot3^3=\frac{27}{4}$

$x:y=1:1\Rightarrow x=y=\frac{3}{2}$

2. x y $x+y=3$ x^4+y^4

$k=3,\ n=4,\ a=1,\ b=1$ (A) x^3+y^3 $(\frac{1}{2})^3\cdot3^4=\frac{81}{8}$

$$x:y=1:1 \Rightarrow x=y=\frac{3}{2}$$

3. x y $x+4y=2$ x^3+y^3

$a=1$, $b=4$, $k=2$, $n=3$ (A) x^3+y^3 $\frac{2^3}{[(\sqrt{1})^3+(\sqrt{4})^3]^2}=\frac{8}{81}$

$$x:y=1:2 \Rightarrow x=\frac{2}{9}\ ,\ y=\frac{4}{9}$$

4. x y z $x+4y+9z=6$ (1) $x^2+y^2+z^2$ (2) $x^3+y^3+z^3$

(1) $a=1,b=4,c=9,k=6$, $n=2$ (A) $x^2+y^2+z^2$ $\frac{6^2}{(1^2+4^2+9^2)^1}=\frac{18}{49}$

$$x:y:z=1:4:9 \Rightarrow x=\frac{3}{49}\ ,\ y=\frac{12}{49}\ ,\ z=\frac{27}{49}$$

(2) $a=1,b=4,c=9,k=6$, $n=3$ (A) $x^3+y^3+z^3$ $\frac{6^3}{(\sqrt{1}^3+\sqrt{4}^3+\sqrt{9}^3)^2}=\frac{1}{6}$

$$x:y:z=1:2:3 \Rightarrow x=\frac{1}{6}\ ,\ y=\frac{1}{3}\ ,\ z=\frac{1}{2}$$

5. $0<\theta<\frac{\pi}{2}$ $\frac{81}{\sin\theta}+\frac{24}{\cos\theta}$

$$[(\sqrt[3]{\frac{81}{\sin\theta}})^3+(\sqrt[3]{\frac{24}{\cos\theta}})^3]^2\cdot[(\sqrt[3]{\sin^2\theta})^3+(\sqrt[3]{\cos^2\theta})^3]^1$$
$$\ge[(\sqrt[3]{\frac{81}{\sin\theta}})^2\cdot\sqrt[3]{\sin^2\theta}+(\sqrt[3]{\frac{24}{\cos\theta}})^2\cdot\sqrt[3]{\cos^2\theta}]^3=(81^{\frac{2}{3}}+24^{\frac{2}{3}})^3=(13\sqrt[3]{9})^3=13^3\cdot 9$$

$\frac{81}{\sin\theta}+\frac{24}{\cos\theta}$ $39\sqrt{13}$

$\sqrt[3]{\frac{81}{\sin\theta}}:\sqrt[3]{\frac{24}{\cos\theta}}=\sqrt[3]{\sin^2\theta}:\sqrt[3]{\cos^2\theta}$ $\sin\theta:\cos\theta=3:2$

6. $0<\theta<\frac{\pi}{2}$ $\frac{1}{\sin^{\frac{1}{2}}\theta}+\frac{32}{\cos^{\frac{1}{2}}\theta}$ ($a>0$, $b>0$)

$a=1$, $b=32$ $\frac{a}{\sin^{\frac{1}{2}}\theta}+\frac{b}{\cos^{\frac{1}{2}}\theta}$ $(a^{\frac{4}{5}}+b^{\frac{4}{5}})^{\frac{5}{4}}=(1^{\frac{4}{5}}+32^{\frac{4}{5}})^{\frac{5}{4}}=17\sqrt[4]{17}$

$$\frac{1}{\sin^{\frac{1}{2}}\theta}+\frac{32}{\cos^{\frac{1}{2}}\theta}$$ $17\sqrt[4]{17}$

$$\sqrt[5]{\frac{1}{\sin^{\frac{1}{2}}\theta}}:\sqrt[5]{\frac{32}{\cos^{\frac{1}{2}}\theta}}=\sqrt[5]{\sin^2\theta}:\sqrt[5]{\cos^2\theta}$$ $\sin\theta:\cos\theta=1:4$

( ).

$0<\theta<\frac{\pi}{2}$ $\frac{a}{\sin^n\theta}+\frac{b}{\cos^n\theta}$($a>0$, $b>0$, $n\in\mathbf{N}$) $(a^{\frac{2}{n+2}}+b^{\frac{2}{n+2}})^{\frac{n+2}{2}}$

$\sin\theta:\cos\theta=\sqrt[n+2]{a}:\sqrt[n+2]{b}$

( ). n $\frac{q}{p}$ $\frac{a}{\sin^n\theta}+\frac{b}{\cos^n\theta}$ $(a^{\frac{2p}{2p+q}}+b^{\frac{2p}{2p+q}})^{\frac{2p+q}{2p}}$

$\sin\theta:\cos\theta=\sqrt[2p+q]{a^p}:\sqrt[2p+q]{b^p}$

( ). $x^k+y^k+z^k=1$ $\frac{a}{x^{\frac{q}{p}}}+\frac{b}{y^{\frac{q}{p}}}+\frac{c}{z^{\frac{q}{p}}}$ ($x>0$, $y>0$,$z>0$, $a>0,\ b>0,c>0$, $k,p,q\in\mathbf{N}$)

$(a^{\frac{pk}{pk+q}}+b^{\frac{pk}{pk+q}}+c^{\frac{pk}{pk+q}})^{\frac{pk+q}{pk}}$ $x:y:z=\sqrt[pk+q]{a^p}:\sqrt[pk+q]{b^p}:\sqrt[pk+q]{c^p}$

( ). ______

( ). ______